Power-Law Entropic Corrections to Newton's Law and Friedmann Equations

A. Sheykhi [1,2] and S. H. Hendi [2,3]
[1] *Department of Physics, Shahid Bahonar University, P.O. Box 76175, Kerman, Iran*
[2] *Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), Maragha, Iran*
[3] *Physics Department, College of Sciences, Yasouj University, Yasouj 75914, Iran*

A possible source of black hole entropy could be the entanglement of quantum fields in and out the horizon. The entanglement entropy of the ground state obeys the area law. However, a correction term proportional to a fractional power of area results when the field is in a superposition of ground and excited states. Inspired by the power-law corrections to entropy and adopting the viewpoint that gravity emerges as an entropic force, we derive modified Newton's law of gravitation as well as the corrections to Friedmann equations. In a different approach, we obtained power-law corrected Friedmann equation by starting from the first law of thermodynamics at apparent horizon of a FRW universe, and assuming that the associated entropy with apparent horizon has a power-law corrected relation. Our study shows a consistency between the obtained results of these two approaches. We also examine the time evolution of the total entropy including the power-law corrected entropy associated with the apparent horizon together with the matter field entropy inside the apparent horizon and show that the generalized second law of thermodynamics is fulfilled in a region enclosed by the apparent horizon.

arXiv:1011.0676v3 [hep-th] 7 Aug 2011

I. INTRODUCTION

Recently, Verlinde [1] demonstrated that gravity can be interpreted as an entropic force caused by the changes in the information associated with the positions of material bodies. In his new proposal, Verlinde obtained successfully the Newton's law of gravitation, the Poisson's equation and Einstein field equations by employing the holographic principle together with the equipartition law of energy. As soon as Verlinde presented his idea, many relevant works about entropic force appeared. For example, Friedmann equations from entropic force have been derived in Refs. [2, 3]. The Newtonian gravity [4], the holographic dark energy [5] and thermodynamics of black holes [6] have been investigated by using the entropic force approach. It has been shown that uncertainty principle may arise in the entropic force paradigm [7]. Other studies on the entropic force, which raised a lot of attention recently, have been carried out in [8].

On the other hand, string theory, as well as the string inspired braneworld scenarios such as RSII model, suggest a modification of Newton's law of gravitation at small distance scales [9, 10]. In addition, there have been considerable works on quantum corrections to some basic physical laws. The loop quantum corrections to the Newton and Coulomb potential have been investigated in some references (see [11] and references therein). Also, corrections to Friedmann equations from loop quantum gravity has been studied in [12].

Inspired by Verlinde's argument and considering the quantum corrections to the area law of the black hole entropy, one is able to derive some physical equations with correction terms. For example, modified Newton's law of gravitation has been studied in [13], while, modified Friedmann equations have been constructed in [14, 15]. In all these cases [13–15] the corrected entropy has the logarithmic term which arises from the inclusion of quantum effects, motivated from the loop quantum gravity and is due to the thermal equilibrium fluctuations and quantum fluctuations [17]. In addition, entropic corrections to Coulomb's law have also been investigated in [16]. Very recently, by considering the quantum corrections to the area law of black hole entropy, the modified forms of Poisson's equation of gravity, MOND theory of gravitation and Einstein field equations were derived using the entropic force interpretation of gravity [18].

In this paper we would like to consider the effects of the power-law correction terms to the entropy on the Newton's law and Friedmann equation. The power-law corrections to entropy appear in dealing with the entanglement of quantum fields in and out the horizon [19]. Indeed, it has been shown that the origin of black hole entropy may lie in the entanglement of quantum fields between inside and outside of the horizon [19]. Since the modes of gravitational fluctuations in a black hole background behave as scalar fields, one is able to compute the entanglement entropy of such a field, by tracing over its degrees of freedom inside a sphere. In this way the authors of [19] showed that the black hole entropy is proportional to the area of the sphere when the field is in its ground state, but a correction term proportional to a fractional power of area results when the field is in a superposition of ground and excited states. For large horizon areas, these corrections are relatively small and the area law is recovered. Applying this power-law corrected entropy, we obtain the corrections to Newton's law as well as modified Friedmann equation by adopting the viewpoint that gravity emerges as an entropic force.

The outline of our paper is as follows. In the next section, we use Verlinde approach to derive Newton's law of gravitation with a correction term resulting from the entanglement of quantum fields in and out the horizon. In section III, we derive the power-law entropy-corrected Friedmann equation of FRW universe by considering gravity as an entropic force. Then, in section IV, we obtain modified Friedmann equation by applying

the first law of thermodynamics at apparent horizon of a FRW universe. In section V we examine to see whether the power-law entropy-area relation together with the matter field entropy inside the apparent horizon will satisfy the generalized second law of thermodynamics. The last section is devoted to conclusions and discussions.

II. ENTROPIC CORRECTION TO NEWTON'S LAW

According to Verlinde's argument, when a test particle moves apart from the holographic screen, the magnitude of the entropic force on this body has the form

$$F\triangle x = T\triangle S, \tag{1}$$

where $\triangle x$ is the displacement of the particle from the holographic screen, while T and $\triangle S$ are the temperature and the entropy change on the screen, respectively.

In Verlinde's discussion, the black hole entropy S plays a significant role. Indeed, the derivation of Newton's law of gravity depends on the entropy-area relationship $S = k_B A/4\ell_p^2$ of black holes in Einsteins gravity, where $A = 4\pi R^2$ represents the area of the horizon and $\ell_p = \sqrt{G\hbar/c^3}$ is the Planck length. However, the area law of black hole entropy can be modified [19]. The corrected entropy takes the form [20]

$$S = \frac{k_B A}{4\ell_p^2}\left[1 - K_\alpha A^{1-\alpha/2}\right], \tag{2}$$

where α is a dimensionless constant whose value is currently under debate, k_B stands for the Boltzmann constant and

$$K_\alpha = \frac{\alpha(4\pi)^{\alpha/2-1}}{(4-\alpha)r_c^{2-\alpha}}, \tag{3}$$

where r_c is the crossover scale. The second term in the above Eq. (2) may be regarded as a power law correction to the area law, resulting from entanglement, when the wave-function of the field is chosen to be a superposition of ground state and exited states.

Considering the power-law correction to entropy, we show that Newton's law of gravitation as well as Friedman equations will be modified accordingly. First of all, we rewrite Eq. (2) in the following form

$$S = k_B\left[\frac{A}{4\ell_p^2} + s(A)\right], \tag{4}$$

where $s(A)$ stands for the correction term in the entropy expression. Suppose we have two masses one a test mass and the other considered as the source with respective masses m and M. Centered around the source mass M, is a spherically symmetric surface $\mathcal{S}$ which will be defined with certain properties that will be made explicit later. To derive the entropic law, the surface $\mathcal{S}$ is between the test mass and the source mass, but the test mass is assumed to be very close to the surface as compared to its reduced Compton wavelength $\lambda_m = \frac{\hbar}{mc}$. When a test mass m is a distance $\triangle x = \eta\lambda_m$ away from the surface $\mathcal{S}$, the entropy of the surface changes by one fundamental unit $\triangle S$ fixed by the discrete spectrum of the area of the surface via the relation

$$\triangle S = \frac{\partial S}{\partial A}\triangle A = k_B\left(\frac{1}{4\ell_p^2} + \frac{\partial s(A)}{\partial A}\right)\triangle A. \tag{5}$$

The energy of the surface $\mathcal{S}$ is identified with the relativistic rest mass of the source mass:

$$E = Mc^2. \tag{6}$$

On the surface $\mathcal{S}$, there live a set of "bytes" of information that scale proportional to the area of the surface so that

$$A = QN, \tag{7}$$

where N represents the number of bytes and Q is a fundamental constant which should be specified later. Assuming the temperature on the surface is T, and then according to the equipartition law of energy [21], the total energy on the surface is

$$E = \frac{1}{2}Nk_B T. \tag{8}$$

Finally, we assume that the force on the particle follows from the generic form of the entropic force governed by the thermodynamic equation

$$F = T\frac{\triangle S}{\triangle x}, \tag{9}$$

where $\triangle S$ is one fundamental unit of entropy when $|\triangle x| = \eta\lambda_m$, and the entropy gradient points radially from the outside of the surface to inside. Note that N is the number of bytes and thus we set $\triangle N = 1$; hence from (7) we have $\triangle A = Q$. Combining Eqs. (5)- (9), we find

$$F = -\frac{Mm}{R^2}\left(\frac{Q^2c^3}{8\pi\hbar\eta\ell_p^2}\right)\left[1 + 4\ell_p^2\frac{\partial s(A)}{\partial A}\right]_{A=4\pi R^2}. \tag{10}$$

This is nothing but the Newton's law of gravitation to the first order provided we define $Q^2 = 8\pi\eta\ell_p^4$. Thus we reach

$$F = -\frac{GMm}{R^2}\left[1 + 4\ell_p^2\frac{\partial s}{\partial A}\right]_{A=4\pi R^2}. \tag{11}$$

Using Eq. (2) we obtain

$$\left(\frac{\partial s}{\partial A}\right)_{A=4\pi R^2} = -\frac{K_\alpha(4-\alpha)}{8\ell_p^2}\left(4\pi R^2\right)^{1-\alpha/2} \tag{12}$$

Substituting Eq. (12) in Eq. (11) we obtain

$$F = -\frac{GMm}{R^2}\left[1 - \frac{K_\alpha}{2}(4-\alpha)\left(4\pi R^2\right)^{1-\alpha/2}\right], \tag{13}$$

Using Eq. (3) the above relation can be rewritten as

$$F = -\frac{GMm}{R^2}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right], \tag{14}$$

This is the power-law correction to the Newton's law of gravitation. When $\alpha = 0$, one recovers the usual Newton's law. Since gravity is an attractive force we should have $F < 0$. This requires

$$1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2} > 0, \tag{15}$$

which can also be rewritten as

$$\alpha < 2\left(\frac{R}{r_c}\right)^{\alpha-2}, \tag{16}$$

As we will see in section V, this condition is also necessary for satisfaction of the generalized second law of thermodynamics for the universe with the power-law corrected entropy.

III. ENTROPIC CORRECTIONS TO FRIEDMANN EQUATIONS

Next, we extend our discussion to the cosmological setup. Assuming the background spacetime to be spatially homogeneous and isotropic which is described by the line element

$$ds^2 = h_{\mu\nu}dx^\mu dx^\nu + R^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{17}$$

where $R = a(t)r$, $x^0 = t, x^1 = r$, the two dimensional metric $h_{\mu\nu}$=diag $(-1, a^2/(1-kr^2))$. Here k denotes the curvature of space with $k = 0, 1, -1$ corresponding to open, flat, and closed universes, respectively. The dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is determined by the relation $h^{\mu\nu}\partial_\mu R \partial_\nu R = 0$. A simple calculation gives the apparent horizon radius for the Friedmann-Robertson-Walker (FRW) universe

$$R = ar = \frac{1}{\sqrt{H^2 + k/a^2}}, \tag{18}$$

where $H = \dot{a}/a$ is the Hubble parameter. We also assume the matter source in the FRW universe is a perfect fluid of mass density ρ and pressure p with stress-energy tensor

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}. \tag{19}$$

Due to the pressure, the total mass $M = \rho V$ in the region enclosed by the boundary $\mathcal{S}$ is no longer conserved, the change in the total mass is equal to the work made by the pressure $dM = -pdV$, which leads to the well-known continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{20}$$

It is instructive to first derive the dynamical equation for Newtonian cosmology. Consider a compact spatial region V with a compact boundary $\mathcal{S}$, which is a sphere with physical radius $R = a(t)r$. Note that here r is a dimensionless quantity which remains constant for any cosmological object partaking in free cosmic expansion. Combining the second law of Newton for the test particle m near the surface with gravitational force (14) we get

$$F = m\ddot{R} = m\ddot{a}r = -\frac{GMm}{R^2}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right], \tag{21}$$

We also assume $\rho = M/V$ is the energy density of the matter inside the the volume $V = \frac{4}{3}\pi a^3 r^3$. Thus, Eq. (21) can be rewritten as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right], \tag{22}$$

This is nothing but the power-law entropy-corrected dynamical equation for Newtonian cosmology. The main difference between this equation and the standard dynamical equation for Newtonian cosmology is that the correction terms now depends explicitly on the radius R. However, we can remove this confusion. Assuming that for Newtonian cosmology the spacetime is Minkowskian with $k = 0$, then we get $R = 1/H$, and we can rewrite Eq. (22) in the form

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left[1 - \frac{\alpha}{2}{r_c}^{\alpha-2}\left(\frac{\dot{a}}{a}\right)^{\alpha-2}\right]. \tag{23}$$

It was argued in [22] that for deriving the Friedmann equations of FRW universe in general relativity, the quantity that produces the acceleration is the active gravitational mass $\mathcal{M}$ [23], rather than the total mass M in the spatial region V. With the entropic correction term, the active gravitational mass $\mathcal{M}$ will also modified as well. On one side, from Eq. (22) with replacing M with $\mathcal{M}$ we have

$$\mathcal{M} = -\frac{\ddot{a}a^2}{G}r^3\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]^{-1}. \tag{24}$$

On the other side, the active gravitational mass is defined as [22]

$$\mathcal{M} = 2\int_V dV\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)u^\mu u^\nu. \tag{25}$$

A simple calculation leads

$$\mathcal{M} = (\rho + 3p)\frac{4\pi}{3}a^3 r^3. \tag{26}$$

Equating Eqs. (24) and (26), we find

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]. \tag{27}$$

Multiplying $\dot{a}a$ on both sides of Eq. (27), and using the continuity equation (20) we reach

$$\frac{d}{dt}(\dot{a}^2) = \frac{8\pi G}{3}\frac{d}{dt}(\rho a^2)\left[1-\frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]. \qquad (28)$$

Integrating of Eq. (28), we find

$$H^2+\frac{k}{a^2} = \frac{8\pi G}{3}\rho\left[1-\frac{\alpha}{2\rho a^2}\left(\frac{r_c}{r}\right)^{\alpha-2}\int\frac{d(\rho a^2)}{a^{\alpha-2}}\right], \qquad (29)$$

where k is a constant of integration. Now, in order to calculate the integral we need to find $\rho = \rho(a)$. Assuming the equation of state parameter $w = p/\rho$ is a constant, the continuity equation (20) can be integrated immediately to give

$$\rho = \rho_0 a^{-3(1+w)}, \qquad (30)$$

where ρ_0 is the present value of the energy density. Inserting relation (30) in Eq. (29), after integration, we obtain

$$H^2+\frac{k}{a^2} = \frac{8\pi G}{3}\rho\left[1-\beta\left(\frac{r_c}{R}\right)^{\alpha-2}\right], \qquad (31)$$

where we have defined

$$\beta = \frac{\alpha}{2}\frac{(3w+1)}{(3w+\alpha-1)}. \qquad (32)$$

Using Eq. (18), we can rewrite Eq. (31) as

$$H^2+\frac{k}{a^2} = \frac{8\pi G}{3}\rho \times\left[1-\beta {r_c}^{\alpha-2}\left(H^2+\frac{k}{a^2}\right)^{\alpha/2-1}\right] \qquad (33)$$

In the absence of the correction terms ($\alpha = 0 = \beta$), one recovers the well-known Friedmann equation in standard cosmology. Let us note that the left hand side of Eq. (33) is always positive thus the right hand side is also positive. This is due to the fact that the right hand side of the usual Friedmann equation is always positive ($\rho > 0$ and $G > 0$) so $H^2 + k/a^2$ should be positive in our case to have a correct $\beta = 0 = \alpha$ limit. This leads to the following condition

$$\beta {r_c}^{\alpha-2}\left(H^2+\frac{k}{a^2}\right)^{\alpha/2-1} < 1. \qquad (34)$$

Eq. (33) can also be written as

$$\left(H^2+\frac{k}{a^2}\right)\left[1-\beta {r_c}^{\alpha-2}\left(H^2+\frac{k}{a^2}\right)^{\alpha/2-1}\right]^{-1} = \frac{8\pi G}{3}\rho. \qquad (35)$$

Taking into account condition (34) we can expand the above equation up to the linear order of β. The result is

$$\left(H^2+\frac{k}{a^2}\right)+\beta {r_c}^{\alpha-2}\left(H^2+\frac{k}{a^2}\right)^{\alpha/2} = \frac{8\pi G}{3}\rho, \qquad (36)$$

where we have neglected $O(\beta^2)$ terms and higher powers of β. This is due to the fact that at the present time $R \gg 1$ and hence $H^2 + k/a^2 \ll 1$ (see the right hand side of standard Friedman equation where $G \sim 10^{-11}$ and $\rho \ll 1$). Indeed for the present time where the apparent horizon area (radius) becomes large, the power-law correction terms to entropy [19] and hence to Friedman equation are relatively small and the usual Friedman equation is recovered. Thus, the corrections make sense only at the early stage of the universe where $a \to 0$. When $a \to 0$, even the higher powers of β should be considered. These correction terms at the early stage of the universe may affect on the number of e-folding during the inflation. However this issue should be examined carefully elsewhere. With expansion of the universe, the power-law entropy-corrected Friedmann equation reduces to the usual Friedman equation.

IV. MODIFIED FRIEDMANN EQUATIONS FROM THE FIRST LAW

In this section, we adopt another approach to derive the entropy-corrected Friedmann equation. Indeed, we are able to derive modified Friedmann equation by applying the first law of thermodynamics at apparent horizon of a FRW universe, with the assumption that the associated entropy with apparent horizon has the power-law corrected form (2). It was already shown that the differential form of the Friedmann equation in the FRW universe can be written in the form of the first law of thermodynamics on the apparent horizon [24]. We follow the method developed in [25]. Throughout this section we set $\hbar = c = k_B = 1$ for simplicity. The associated temperature with the apparent horizon can be defined as [26]

$$T = \frac{\kappa}{2\pi} = -\frac{1}{2\pi R}\left(1-\frac{\dot{R}}{2HR}\right). \qquad (37)$$

where κ is the surface gravity. When $\dot{R} \leq 2HR$, the temperature becomes negative $T \leq 0$. Physically it is not easy to accept the negative temperature. In this case the temperature on the apparent horizon should be defined as $T = |\kappa|/2\pi$. The work density is obtained as [27]

$$W = \frac{1}{2}(\rho - p). \qquad (38)$$

The work density term is regarded as the work done by the change of the apparent horizon. We also assume the first law of thermodynamics on the apparent horizon is satisfied and has the form

$$dE = T_h dS_h + W dV, \qquad (39)$$

where S_h is the power-law corrected entropy associated with the apparent horizon which has the form (2). Suppose $E = \rho V$ is the total energy content of the universe inside a 3-sphere of radius R, where $V = \frac{4\pi}{3}R^3$ is the volume enveloped by 3-dimensional sphere with the area of apparent horizon $A = 4\pi R^2$. Taking differential form of the relation $E = \frac{4\pi}{3}\rho R^3$ for the total matter and energy inside the apparent horizon, and using the continuity equation (20), we get

$$dE = 4\pi\rho R^2 dR - 4\pi H R^3(\rho + p)dt. \tag{40}$$

Taking differential form of the corrected entropy (2), we have

$$dS_h = \frac{2\pi R}{G}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]dR. \tag{41}$$

Inserting Eqs. (37), (38), (40) and (41) in the first law (39), we can get the differential form of the modified Friedmann equation

$$\frac{1}{4\pi G}\frac{dR}{R^3}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right] = H(\rho + p)dt. \tag{42}$$

Using the continuity equation (20), we can rewrite it as

$$-\frac{2}{R^3}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]dR = \frac{8\pi G}{3}d\rho. \tag{43}$$

Integrating (43) yields

$$\frac{1}{R^2} - \frac{r_c^{\alpha-2}}{R^\alpha} = \frac{8\pi G}{3}\rho + C, \tag{44}$$

where C is an integration constant to be determined later. Substituting R from Eq. (18) we obtain entropy-corrected Friedmann equation

$$H^2 + \frac{k}{a^2} - r_c^{\alpha-2}\left(H^2 + \frac{k}{a^2}\right)^{\alpha/2} = \frac{8\pi G}{3}\rho + C. \tag{45}$$

The constant C can be determined by taking the $\alpha \to 0$ limit of the above expression. In this limit Eq. (45) reduces to the usual Friedmann equation provided $C = -r_c^{-2}$. Thus we reach

$$H^2 + \frac{k}{a^2} - r_c^{-2}\left[r_c^\alpha\left(H^2 + \frac{k}{a^2}\right)^{\alpha/2} - 1\right] = \frac{8\pi G}{3}\rho. \tag{46}$$

This is the power-law entropy corrected Friedmann equation derived using the first law on the apparent horizon. To show the consistency between the result of this section with Eq. (36) derived in the previous section, let us note that Eq. (36) in the previous section was derived for the late time where the term $O(\beta^2)$ can be neglected and thus we do not expect to be exactly the same as the result obtained in this section which is valid for all epoch of the universe. However, if one absorbs, in Eq. (45), the constant C in ρ, then one can rewrite Eq. (45) as

$$H^2 + \frac{k}{a^2} - r_c^{\alpha-2}\left(H^2 + \frac{k}{a^2}\right)^{\alpha/2} = \frac{8\pi G}{3}\rho, \tag{47}$$

which is consistent with Eq. (36) derived using the entropic force approach in the previous section provided one takes $\beta = -1$, which can be translated into

$$w = \frac{2 - 3\alpha}{3\alpha + 6}. \tag{48}$$

For $\alpha > 2$, the above relation leads to $w < -1/3$. Two points should be considered here carefully. First, relation (48) was derived for $\beta = -1$, thus it does not have $\alpha = 0$ limit, since in this case ($\alpha = 0$), from definition (32) we have $\beta = 0$, which is in contradiction with condition $\beta = -1$. Second, relation (48) appears when we want to show the consistency between modified Friedman equation derived from two methods. In the absence of correction terms ($\alpha = 0 = \beta$) the obtained Friedman equations from two different methods, namely Eqs. (36) and (46) exactly coincide regardless of the value of w. This indicates that for usual Friedmann equation the condition (48) is relaxed and hence w can have any arbitrary value in standard cosmology.

It is also notable to mention that Eq. (46) is consistent with the result obtained in [28]. However, our derivation is quite different from [28]. Let us stress the difference between our derivation in this section and [28]. First of all, the authors of [28] have derived modified Friedmann equations by applying the first law of thermodynamics, $TdS = -dE$, to the apparent horizon of a FRW universe with the assumption that the apparent horizon has corrected-entropy like (2). It is worthy to note that the notation dE in [28] is quite different from the same we used in this section. In [28], $-dE$ is actually just the heat flux crossing the apparent horizon within an infinitesimal internal of time dt. But, here dE is change in the the matter energy inside the apparent horizon. Besides, in [28] the apparent horizon radius R has been assumed to be fixed. But, here, the apparent horizon radius changes with time. This is the reason why we have included the term WdV in the first law (39). Indeed, the term $4\pi R^2\rho dR$ in Eq. (40) contributes to the work term, while this term is absent in dE of [28]. This is consistent with the fact that in thermodynamics the work is done when the volume of the system is changed.

V. GENERALIZED SECOND LAW OF THERMODYNAMICS

Finally, we investigate the validity of the generalized second law of thermodynamics for the power-law entropy corrected Friedmann equations in a region enclosed by the apparent horizon. Our method here differs from that of Ref. [20], in that they studied the generalized second law along with either Clausius relation or the equipartition law of energy, while we apply the first law of thermodynamics (39). The difference between our method and Ref. [28] was also explained in the last paragraph of the previous section. Substituting relation (18) in modified

Friedmann Eq. (46) we find

$$\frac{1}{R^2}-\frac{r_c^{\alpha-2}}{R^\alpha}+r_c^{\alpha-2}=\frac{8\pi G}{3}\rho \tag{49}$$

Differentiating Eq. (49) with respect to the cosmic time, after using the continuity Eq. (20), we get

$$\dot{R}=4\pi GHR^3(\rho+p)\left[1-\frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]^{-1}. \tag{50}$$

Next, we calculate $T_h\dot{S_h}$. Using Eq. (41) we find

$$T_h\dot{S_h}=\frac{1}{2\pi R}\left(1-\frac{\dot{R}}{2HR}\right)\times\frac{2\pi R}{G}\left[1-\frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]\dot{R} \tag{51}$$

After some simplification and using Eq. (50) we obtain

$$T_h\dot{S_h}=4\pi HR^3(\rho+p)\left(1-\frac{\dot{R}}{2HR}\right). \tag{52}$$

In the accelerating universe the dominant energy condition may violate, $\rho+p<0$, indicating that the second law of thermodynamics ,$\dot{S_h}\geq 0$, does not hold. However, as we will see below the generalized second law of thermodynamics, $\dot{S_h}+\dot{S_m}\geq 0$, is still fulfilled throughout the history of the universe. From the Gibbs equation we have [31]

$$T_m dS_m = d(\rho V)+pdV = Vd\rho+(\rho+p)dV, \tag{53}$$

where T_m and S_m are, respectively, the temperature and the entropy of the matter fields inside the apparent horizon. We limit ourselves to the assumption that the thermal system bounded by the apparent horizon remains in equilibrium so that the temperature of the system must be uniform and the same as the temperature of its boundary. This requires that the temperature T_m of the energy inside the apparent horizon should be in equilibrium with the temperature T_h associated with the apparent horizon, so we have $T_m = T_h$ [31]. This expression holds in the local equilibrium hypothesis. If the temperature of the fluid differs much from that of the horizon, there will be spontaneous heat flow between the horizon and the fluid and the local equilibrium hypothesis will no longer hold. Therefore from the Gibbs equation (53) we can obtain

$$T_h\dot{S_m}=4\pi R^2\dot{R}(\rho+p)-4\pi R^3H(\rho+p). \tag{54}$$

To check the generalized second law of thermodynamics, we have to examine the evolution of the total entropy S_h+S_m. Adding equations (52) and (54), we get

$$T_h(\dot{S_h}+\dot{S_m})=2\pi R^2(\rho+p)\dot{R}=\frac{A}{2}(\rho+p)\dot{R}. \tag{55}$$

where $A=4\pi R^2$ is the apparent horizon area. Substituting $\dot{R}$ from Eq. (50) into (55) we find

$$T_h(\dot{S_h}+\dot{S_m})=2\pi GAHR^3(\rho+p)^2\left[1-\frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]^{-1}. \tag{56}$$

As we argued after Eq. (14) the expression in the bracket of Eq. (56) is always positive i.e.,

$$\left[1-\frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]>0. \tag{57}$$

Thus the right hand side of Eq. (56) cannot be negative throughout the history of the universe, which means that $\dot{S_h}+\dot{S_m}\geq 0$ always holds. This implies that for a universe with power-law entropy corrected relation the generalized second law of thermodynamics is fulfilled in a region enclosed by the apparent horizon. Note that if we identify the crossover scale r_c with the present value of the apparent horizon, i.e., $r_c=R$, then the condition (57) reduces to $\alpha<2$, which is consistent with the result obtained in [20, 28].

VI. CONCLUSIONS AND DISCUSSIONS

It was argued that a possible source of black hole entropy could be the entanglement of quantum fields in and out the horizon [19]. The entanglement entropy of the ground state of field obeys the well-known area law. However, the power-law correction to the area law appears when the wave-function of the quantum field is chosen to be a superposition of ground state and exited state [19]. Indeed, the excited states contribute to the correction, and more excitations produce more deviation from the area law [29, 30]. Therefore, the correction terms are more significant for higher excitations.

Motivated by the power-law corrected entropy and adopting the viewpoint that gravity emerges as an entropic force, we derived modified Newton's law of gravitation as well as power-law correction to Friedmann equations. We found that the correction term for Friedmann equation falls off rapidly with apparent horizon radius and can be comparable to the first term only when the scale factor a is very small. Thus the corrections make sense only at early stage of the universe. When the universe becomes large, the power-law entropy-corrected Friedmann equation reduces to the standard Friedman equation. This can be understood easily. At late time where a is large, i.e., at low energies, it is difficult to excite the modes and hence, the ground state modes contribute to most of the entanglement entropy. However, at the early stage, i.e., at high energies, a large number of field modes can be excited and contribute significantly to the correction causing deviation from the area law and hence deviation from the standard Friedmann equation.

We also derived modified Friedmann equation from different approach. Starting from the first law of thermodynamics at apparent horizon of a FRW universe, and assuming that the associated entropy with apparent horizon has power-law corrected form (2), we obtained modified Friedmann equation. We find out that the derived modified equations from these two different approaches (entropic force approach and first law approach) can be

consistent provided the equation of state parameter satisfies in condition (48). However, in the absence of the correction terms ($\alpha = 0 = \beta$) the obtained Friedman equations from two different methods, namely Eqs. (36) and (46) exactly coincide regradless of the value of w. This indicates that for usual Friedmann equation the condition (48) is relaxed and hence w can have any arbitrary value in standard cosmology.

Finally, we investigated the validity of the generalized second law of thermodynamics for the FRW universe with any spatial curvature. We have shown that, when thermal system bounded by the apparent horizon remains in equilibrium with its boundary such that $T_m = T_h$, the generalized second law of thermodynamics is fulfilled in a region enclosed by the apparent horizon. The results obtained here for power-law corrected entropy area relation further supports the thermodynamical interpretation of gravity and provides more confidence on the profound connection between gravity and thermodynamics.

Acknowledgments

We thank the anonymous referee for constructive and valuable comments. This work has been supported by Research Institute for Astronomy and Astrophysics of Maragha.

---

[1] E. P. Verlinde, arXiv:1001.0785.
[2] F. W. Shu and Y. G. Gong, arXiv:1001.3237.
[3] R. G. Cai, L. M. Cao and N. Ohta, Phys. Rev. D **81**, 061501 (2010).
[4] L. Smolin, arXiv:1001.3668.
[5] M. Li and Y. Wang, Phys. Lett. B **687**, 243 (2010).
[6] Y. Tian and X. Wu, Phys. Rev. D **81**, 104013 (2010); Y. S. Myung, arXiv:1002.0871.
[7] I. V. Vancea and M. A. Santos, arXiv:1002.2454.
[8] Y. X. Liu, Y. Q. Wang and S.W. Wei, Class. Quantum Grav. **27**, 185002 (2010);
V. V. Kiselev and S. A. Timofeev, Mod. Phys. Lett. A **25**, 2223 (2010);
R. A. Konoplya, Eur. Phys. J. C **69**, 555 (2010);
R. Banerjee and B. R. Majhi. Phys. Rev. D **81**, 124006 (2010);
P. Nicolini, Phys. Rev. D **82**, 044030 (2010);
C. Gao, Phys. Rev. D **81**, 087306 (2010);
Y. S. Myung and Y. W. Kim, Phys. Rev. D **81**, 105012 (2010);
H. Wei, Phys. Lett. B **692**, 167 (2010);
Y. Ling and J.P. Wu, JCAP **1008**, 017 (2010);
D. A. Easson, P. H. Frampton and G. F. Smoot, arXiv:1002.4278;
D. A. Easson, P. H. Frampton and G. F. Smoot, arXiv:1003.1528;
S. W. Wei, Y. X. Liu and Y. Q. Wang, arXiv:1001.5238.
[9] J. Polchinski, *String Theory Vol I and II*, Cambridge University Press, (1998).
[10] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999);
Shiromizu et al, Phys. Rev. D **62**, 024012 (2000);
D. K. Park and S. Tamaryan, Phys. Lett. B **554**, 92 (2003);
M. Azam et al, Phys. Rev. D **77**, 101101 (2008).
[11] J. F. Donoghue, Phys. Rev. D **50**(6), 3874 (1994);
J. F. Donoghue, Phys. Rev. Lett., **72**(19), 2996 (1994);
A. Akhundov et al, Phys. Lett. B **395**, 16 (1997);
B. F. L. Ward, Mod. Phys. Lett. A **17**, 2371 (2002);
B. F. L. Ward, Int. J. Mod. Phys. A **20**, 3502 (2005);
G. G. Kirilin and I. B. Khriplovich, J. Exp. Theor. Phys. **95**, 981 (2002);
S. Faller, Phys. Rev. D **77**, 124039 (2008).
[12] V. Taveras, Phys. Rev. D **78**, 064072 (2008).
[13] L. Modesto and A. Randono, arXiv:1003.1998.
[14] A. Sheykhi, Phys. Rev. D **81**, 104011 (2010).
[15] B. Liu, Y. C. Dai, X. R. Hu and J. B. Deng, Mod. Phys. Lett. A **26**, 489 (2011).
[16] A. Sheykhi and S. H. Hendi, arXiv:1009.5561.
[17] C. Rovelli, Phys. Rev. Lett. **77**, 3288 (1996);
A. Ashtekar, J. Baez, A. Corichi and K. Krasnov, Phys. Rev. Lett. **80**, 904 (1998).
[18] S. H. Hendi and A. Sheykhi, Phys. Rev. D **83** (2011) 084012.
[19] Saurya Das, S. Shankaranarayanan and Sourav Sur, Phys. Rev. D, **77**, 064013 (2008).
[20] N. Radicella and D. Pavon, Phys. Lett. B **691**, 121 (2010).
[21] T. Padmanabhan, Mod. Phys. Lett. A **25**, 1129 (2010);
T. Padmanabhan, Phys. Rev. D **81**, 124040 (2010).
[22] R.G. Cai, L. M. Cao and N. Ohta, Phys. Rev. D **81**, 061501(R) (2010).
[23] T. Padmanabhan, Class. Quantum Grav. **21**, 4485 (2004).
[24] R.G. Cai, S.P. Kim, JHEP **02**, 050 (2005);
A. Sheykhi, B. Wang and R. G. Cai, Nucl. Phys. **B779**, 1 (2007);
A. Sheykhi, B. Wang and R. G. Cai, Phys. Rev. D **76**, 023515 (2007);
R. G. Cai and L. M. Cao, Nucl. Phys. B **785**, 135 (2007);
A. Sheykhi, JCAP **05**, 019 (2009).
[25] A. Sheykhi, Eur. Phys. J. C **69**, 265 (2010).
[26] A. Sheykhi, B. Wang, Phys. Lett. B **678**, 434 (2009);
A. Sheykhi, B. Wang, Mod. Phys. Lett. A **25**, 1199 (2010).
[27] S. A. Hayward, S.Mukohyana, andM. C. Ashworth, Phys. Lett. A **256**, 347 (1999);
S. A. Hayward, Class. Quant. Grav. **15**, 3147 (1998).
[28] K. Karami, N. Sahraei and S. Ghaffari, arXiv:1009.3833.
[29] S. Das, S. Shankaranarayanan and S. Sur, arXiv:1002.1129.
[30] S. Das, S. Shankaranarayanan and S. Sur, arXiv:0806.0402.
[31] G. Izquierdo and D. Pavon, Phys. Lett. B **633** (2006) 420.